

Alberta Electric System Operator

AUC Proceeding

Milner Power Complaint

Two-Bus Test System Report on Loss Factor Methodology

Executive Summary

During the AUC meeting held in Calgary on February 3, It was suggested by Milner representatives that unless the methodologies gave the "correct"¹ results and signals with a two bus system, then, the methodologies would not be valid. A brief study was undertaken to evaluate the results obtained using the AESO current loss factor methodology, i.e., "50% Area Load Methodology" and the Incremental Loss Factor Methodology proposed by Milner representatives.

The findings of the evaluation are that the AESO current methodology "gives correct results" in a two bus system. In addition, analysis shows, the Incremental Loss Factor Methodology proposed by Milner gave "correct" results for only the single scenario with similar sized generators at each end of the test system. Further, the Incremental Loss Factor Methodology proposed by Milner fails to meet important criteria necessary for a reasonable loss factor methodology.

1. Introduction

During the AUC meeting held in Calgary on February 3, it was suggested by Milner representatives that unless the methodologies gave the "correct" results and signals with a two bus system, then, the methodologies would not be valid. The objective of this document is to present the findings of a recent investigation carried out using a two bus system, that compares results using the current loss factor methodology (50% Area Load) with the Incremental Loss Factor Methodology (ILF) as proposed by Milner representatives.

Development of loss factors for the AESO is a complicated process. The discussion hereinafter refers only to the initial part of the process in which load flow based loss factors are determined. It is the part of the process where the methodology adopted could have a major impact on the loss factors that are assigned to generators, exports and DOS Loads. In the discussion that

¹ The term 'correct' was used by the Milner Representatives during the Feb 3 AUC meeting

follows, the term "raw loss factor" is used to define the loss factors that are determined from each load flow based on the methodology used. As each methodology can result in significantly different "raw loss factors" for each unit, a "correction factor" (either positive or negative) is added to the "raw loss factor" for each generator such that the sum of the products of the resultant loss factor times the generator output of the load flow recovers all of the losses of the load flow. The loss factor with correction applied is referred to hereinafter as "load flow loss factor".

During the AUC meeting, the Milner representatives dismissed the magnitude of the correction as being unimportant. However, the correction factor has been included in these discussions to obtain a comparison of the two methods on the same basis.

2. Test System

A two bus network was simulated as shown in Figure 1. The branch connecting the two buses is intended to represent the losses associated with a transmission system between local generation and load at bus 1 and local generation and load at bus 2. In previous AESO consultations², Milner representatives has indicated that in a two bus example, generation at bus 2 would be large, representing a "deep" system. To provide broader analysis and to account for a deep system scenario, this test system's load and generation were modelled at both busses. The generation and voltage at bus 1 were kept constant at 100 MW and 1.0 p.u. respectively. Load at bus 1 was varied from 0 to 200 MW resulting in flows on the connecting branch ranging from 100 MW bus 1 to bus 2 to 100 MW bus 2 to bus 1. In the "deep" system scenario, the generation at bus 2 was kept constant at 10,000 MW. To demonstrate that " *the methodologies gave the correct results and signals with a two bus system*" for other scenarios, alternative scenarios with generation at bus 2 of 200 MW and generation at bus 1 of 104 MW were also evaluated. The load at bus 2 was adjusted to balance the system. For the scenario with 104 MW of generation at bus 2, the generators are approximately the same size but for the load range of zero to 200 MW at bus 1, the balancing load at bus 2, would not be negative.

² <http://www.aeso.ca/downloads/TransmissionLossFactorMethodology.pdf>

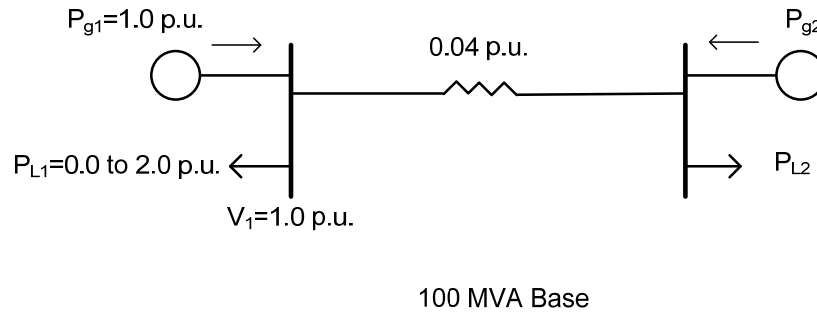


Figure 1: Two Bus Test System

The scenarios studied are summarized as follows:

		Generation (MW)	Load (MW)
Scenario 1	Bus 1	100	0-200
	Bus 2	104	200-0
Scenario 2	Bus 1	100	0-200
	Bus 2	200	296-96
Scenario 3	Bus 1	100	0-200
	Bus 2	10000	10,096-9,896

3. Results with Current Loss Factor Methodology

3.1. Principle

The principle associated with the current methodology, is that the loss factor is based on the change in losses (dl) for a given change in generation (dg) at the bus for which the loss factor is being calculated. Total system load is adjusted (dL) to compensate for the change in generation and losses. All loads are adjusted by a common factor. The loss factor (denoted raw loss factor) at the bus is set equal to one half of the ratio dl/dg . The loss factor for each generator is multiplied by the output of each generator and the products are summed to establish the total losses that would be recovered by the raw loss factors themselves. A constant correction factor (denoted load flow correction factor) is added to the raw loss factor for every generator to obtain a load flow loss factor for each generator. The load flow correction factor is selected such that the sum of the products of the adjusted loss factor and generator output at each bus is equal to the total system losses.

3.2. Calculation Procedure

The loss factors were calculated using a MathCAD worksheet and the results transferred to an Excel spreadsheet for analysis and plotting. The calculation method from the MathCAD worksheet is given in Appendix A. The mathematical implementation of the methodology as used in the spreadsheet is not an exact duplication of the implementation as used in the AESO 50% area load methodology. The latter utilizes the corrected "R" matrix approach as described in [1]. The MathCAD equations are sufficient to demonstrate the principles of the 50% Area Load Methodology.

3.3. Results

The resultant load flow loss factors for the generators at buses 1 and 2 are shown in Figure 2 and Figure 3 respectively. The locational signal for bus1 (equal to the difference in loss factors between bus 1 and bus 2) is shown in Figure 4. Observations of note are:

- a. The maximum load flow loss factor at bus 1 is 4% and occurs when all of its output power flows through the interconnecting circuit. The corresponding load flow loss factor at bus 2 is zero. Both loss factors are independent of the magnitude of generation and load at bus 2. The two loss factors at this loading condition are consistent with the generator at bus 1 causing all of the losses on the system.
- b. The load flow loss factors for both generators are zero when the total losses for the system are zero as occurs when the flow across the connecting circuit is zero.
- c. When power is flowing from bus 2 to bus 1 signalling a need for generation at bus 1, the unit at bus 1 receives a credit for most of the loading conditions simulated and the unit at bus 2 receives a charge. The size of the credits and charges at 1.0 p.u. flow from bus 2 to 1 is a function of the size of the unit at bus 2 and the corresponding load at bus 2. As both generators can be considered to be supplying both loads, they are both contributing to system losses. For the condition where both generators are virtually the same size (100MW vs. 104MW) all of the load (200MW) is located at bus 1. The unit at bus 2 is supplying 50% of the load at bus 1 but can be assumed to cause all of the losses. A charge of close to 4% for unit 2 and essentially no charge for unit 1 is logical in this case. This is effectively the reverse of the situation described in observation "a" above.
- d. In dispatching generation through Alberta's power pool, a generator can be said to supply power to all system loads, not just to local loads. With this in mind the following observation is made. With 100 MW flow from bus 2 to bus 1, (200 MW of generation at bus 2 and 200 MW of load at bus 1 with a corresponding 100 MW of load at bus 2), the bus 2 generator can be assumed to be supplying a part of the bus 1 load. Similarly, bus 1 generation can be assumed to be supplying part of bus 2 load. With percentage based on generator output, the generator at bus 2 would be supplying a greater percentage of

the load at bus 1 (66%). The generator at bus 1 therefore reduces the power supplied to bus 1 at the same time increasing power to bus 2. The increase in power to bus 2 reduces system losses and should result in a reduction in loss charges or credit to the unit at bus 1. The increase in generator 2 contribution to the load at bus 1 increases losses so the generator at bus 2 should be penalized more than the 100 MW unit. However as the unit size is double, its loss factor should be greater than 50% of the loss factor for a 100 MW unit. A loss factor credit of slightly greater than 1% for bus 1 and a loss factor charge of slightly less than 3% as seen in Figure 2 and Figure 3 respectively follow an appropriate trend.

- e. With 10,000 MW of generation at bus 2 along with a corresponding large load, similar reasoning to "d" above can be applied and taken to an extreme. The large unit at bus 2 supplies almost all of the 200 MW load at bus 1 and the 100 MW unit at bus 1 supplies no power to bus 1, reducing system losses. The unit at bus 1 should receive a credit but the credit should not exceed total system losses. The 4% credit shown in Figure 2 is appropriate for this case.
- f. The locational signal for bus 1 as shown in Figure 1 is defined as the difference in loss factors between bus 1 and bus 2. It is independent of size of unit at bus 2. The locational signal should only depend on the net to grid injections at each bus.
- g. The locational signal is zero if the circuit loading is zero, which is logical in that adding generation at either bus with a corresponding increase in load at both buses will impact losses in the same fashion.

The constant correction factor that was added to the raw loss factors to achieve the load flow loss factors of Figure 2 and Figure 3 is shown in Figure 5. Observations of note are:

- a. The largest correction factor required occurred with 200 MW of load at bus 1 and 200 MW of generation at bus 2.
- b. The magnitude of the largest correction factor is small. The largest correction factor is .033% or .83% of the maximum losses.
- c. The correction factor at zero power transfer is zero.

Loss factor volatility was determined numerically for a small change in circuit resistance. The volatility factor defined as the resultant change in loss factor divided by the change in circuit resistance is shown in Figure 6. Observation of note are:

- a. The shapes of the loss factor volatility curves are similar to the shapes of the loss factor curves
- b. The loss factor volatility established by circuit resistance is directly proportional to the generator loss factor

Loss factor volatility for a small change in load at bus 1 is shown in Figure 7. Observation of note are:

- a. The relationship between volatility and power transfer is almost linear.
- b. The volatility appears to be equal to the negative of the derivative of the loss factor curves shown in Figure 2 with respect to power flow between bus 1 and bus 2.
- c. The largest volatility occurs when the generators at both buses are similar in size.

Loss factor volatility for a small change in load at bus 1 is shown in Figure 8. Observation of note are:

- a. The largest volatility for the conditions studied again occurs when the generator at bus 2 is large.
- b. Volatility when all output from unit 1 flows to bus 2 is independent of the size of unit at bus 2.

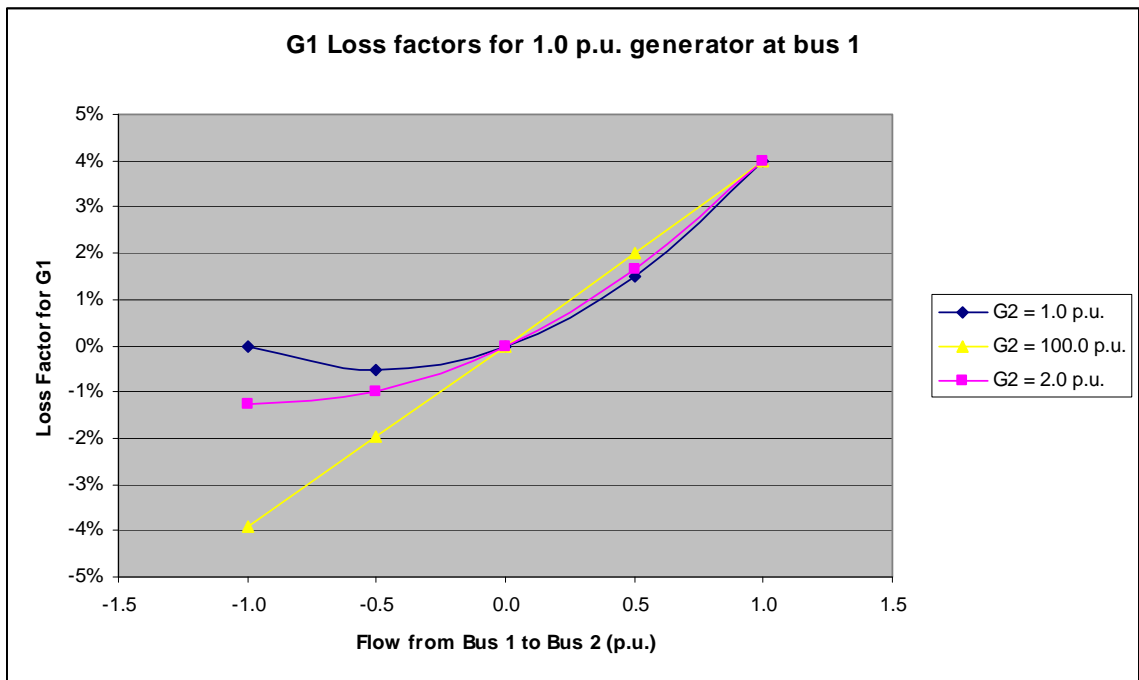


Figure 2: Loss Factors for G1, 50% Area Load Methodology

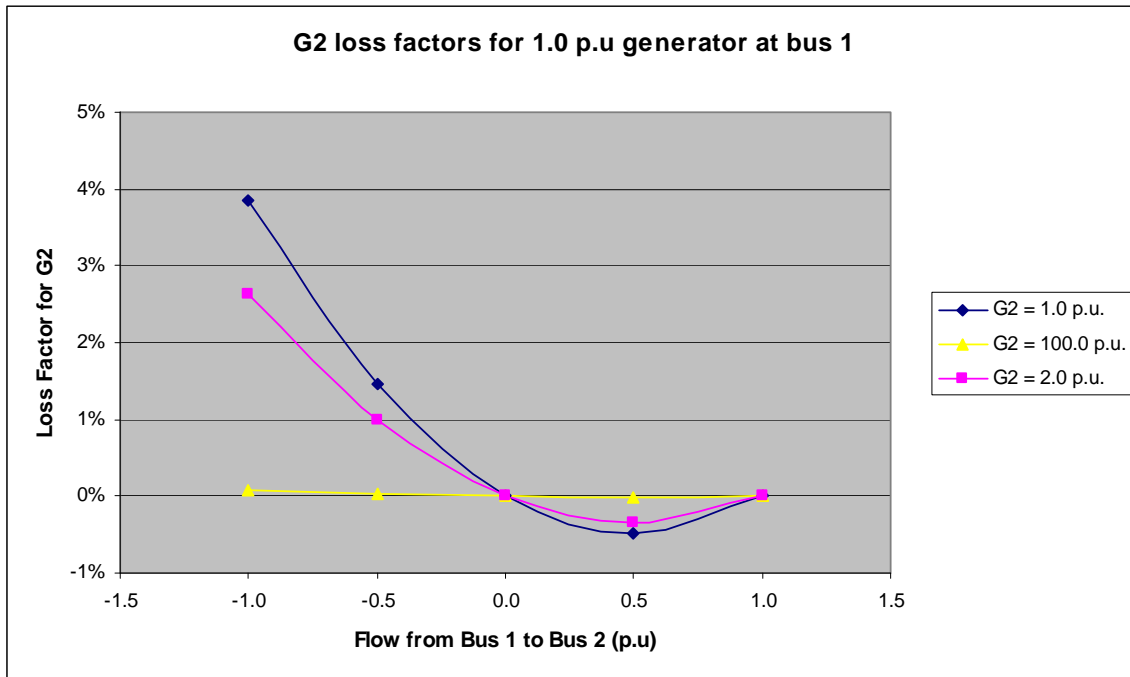


Figure 3: Loss Factors for G2, 50% Area Load Methodology

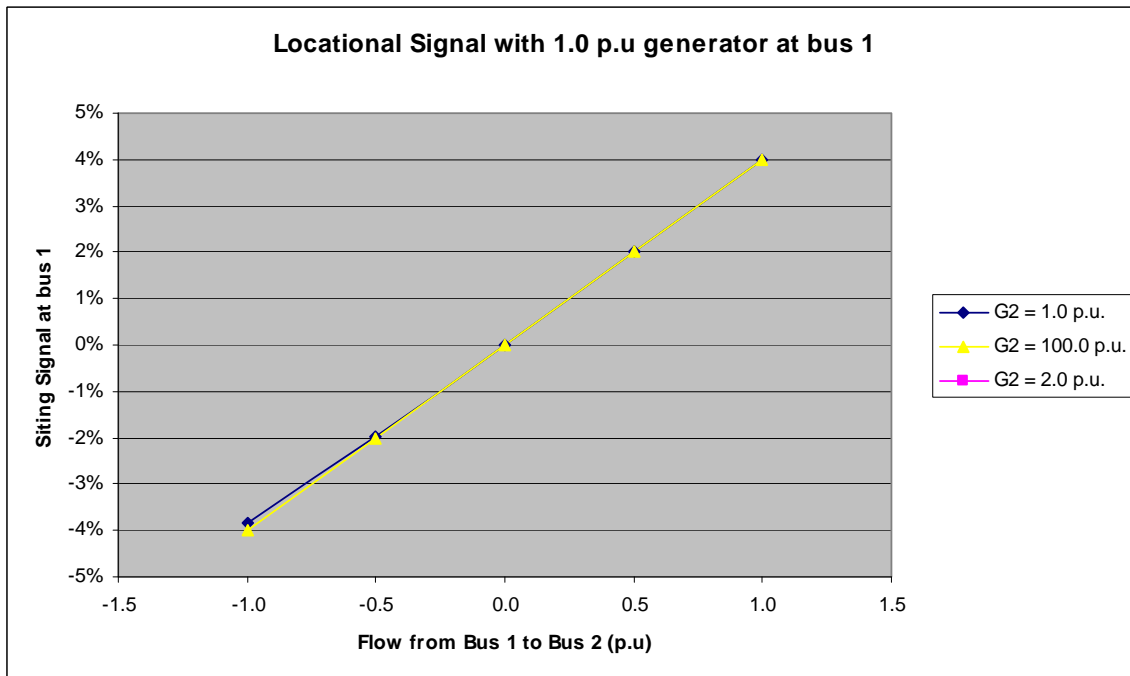


Figure 4: Locational Signal Bus 1 to Bus 2, 50% Area Load Methodology

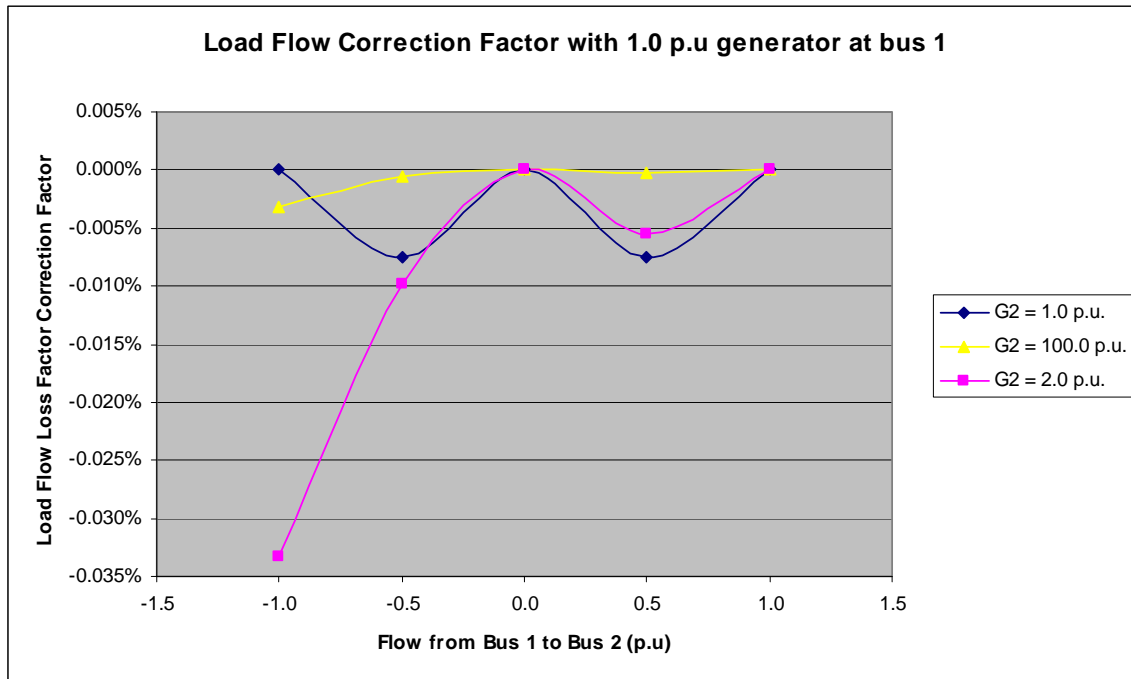


Figure 5: Load Flow Correction Factor, 50% Area Load Methodology

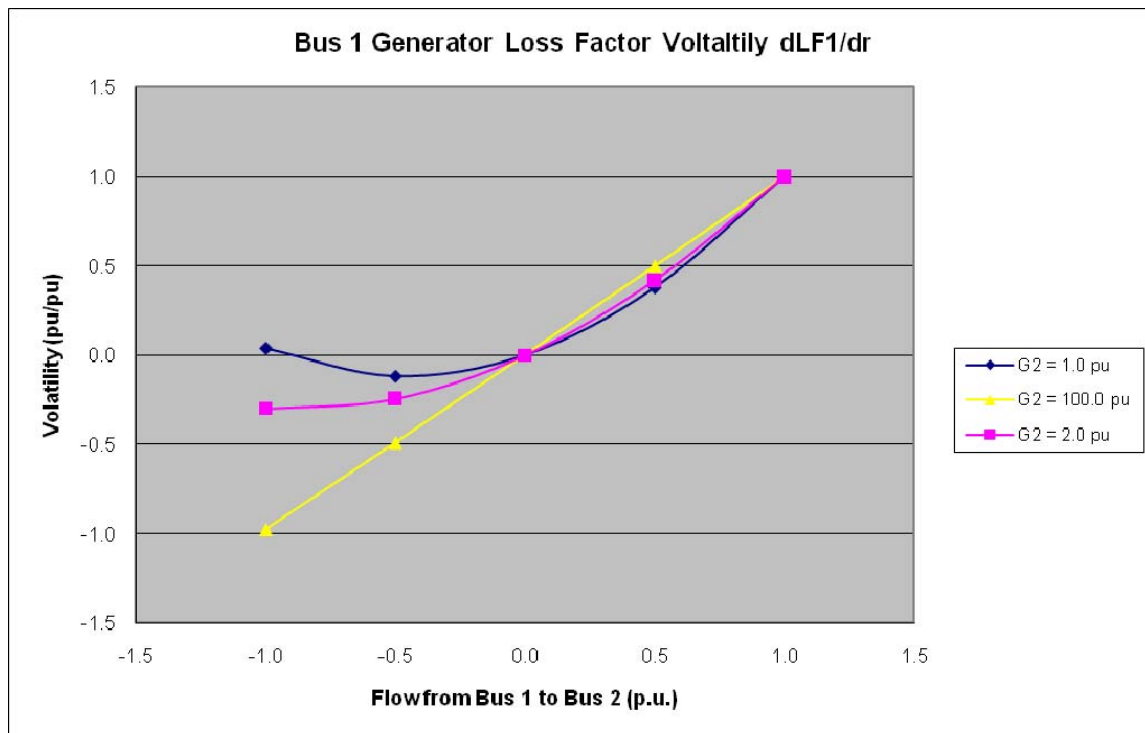


Figure 6: Bus 1 Volatility Based on Resistance Changes , 50% Area Load Methodology

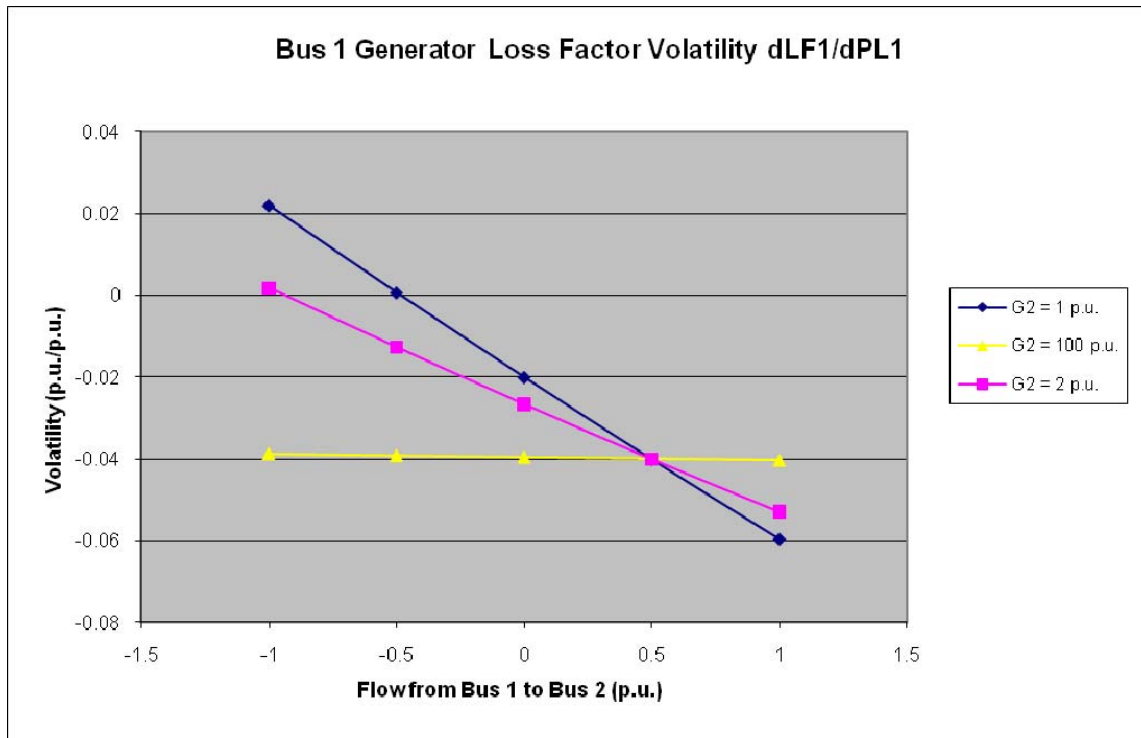


Figure 7: Bus 1 Volatility Based on Bus 1 Load Changes , 50% Area Load Methodology

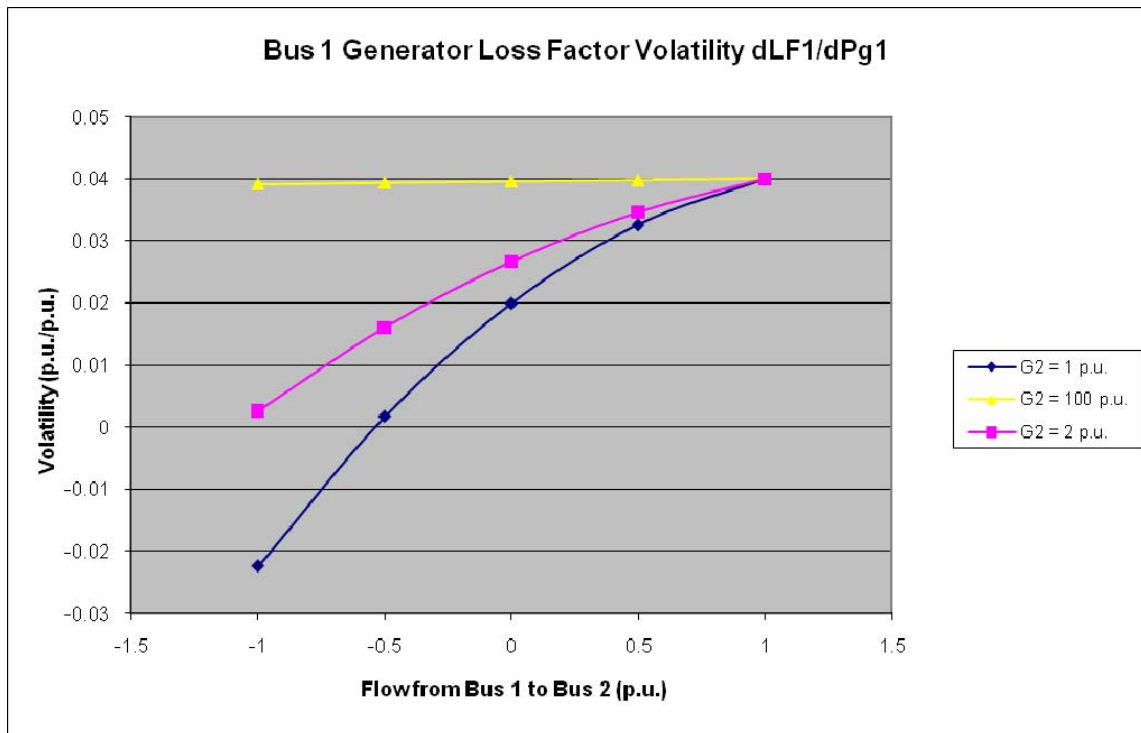


Figure 8: Bus 1 Volatility Based on Bus 1 Generation Changes , 50% Area Load Methodology

4. Results with Incremental Loss Factor Methodology

4.1. Principle

The principle associated with the incremental loss factor methodology is that generator raw loss factors are based on the difference in losses between the condition with the generator in service and the generator out of service or dispatched at zero MW. In the calculations that follow, the losses were first calculated with the output of the generator unit 1 set to 1.0 p.u. and the output of the generator unit 2 set to 1.04, 2.0 or 100 p.u. per the test case. The losses were then recalculated with unit 1 output reduced to 0.0 p.u. for loss factors associated with bus 1 and with unit 2 output reduced to zero for loss factors associated with bus 2. To accommodate the generation and resultant loss change, loads at both buses were reduced by a common factor. The unadjusted raw loss factor was set equal to the change in losses divided by the initial generator output. Similar to the current methodology, both loss factors were adjusted with a common correction factor added to the raw loss factors, such that the resultant load flow loss factors recovered all of the system losses.

4.2. Calculation Procedure

The loss factors were calculated using a MathCAD worksheet and the results transferred to an Excel spreadsheet for analysis and plotting. The calculation method from the MathCAD worksheet is given in Appendix B. The mathematical implementation of the methodology as used in the spreadsheet is not an exact duplication of the implementation as used in the AESO loss factor methodology evaluation [1]. The latter utilized a corrected "R" matrix approach as described in [1]. The MathCAD equations however are sufficient to demonstrate the principles of the Incremental Loss Factor Methodology.

4.3. Results

The resultant load flow loss factors for the generators at buses 1 and 2 are shown in Figure 9 and Figure 10 respectively. The locational signal for bus is shown in Figure 11. Observations of note are:

- a. The maximum load flow loss factor charge at bus 1 is the same as the current methodology, i.e. 4% when the load at bus 1 is zero and all output from generator 1 flows to bus 2. Generator 1 is responsible for all of the losses and hence has a loss factor charge equal to total system losses.
- b. The results when both generators are the same size are the same as the current methodology for all power transfer levels

- c. Loss factors at bus 1 provide larger credits with flows from bus 2 to bus 1, especially if the unit at bus 2 is large, where credits could be as high as 12%
- d. Loss factors at zero transfer are not zero for conditions where the two units are not the same capacity. The impact of this is that generator 2 would be charged for losses even if there are no losses on the system and generator 1 would receive a credit. In the 'deep' system scenario, with a large generator at bus 2, the generator 1 credit could be as large as 4% of its output, even though total system losses are zero.
- e. The locational signal shown in Figure 11 indicates a preference to install new generation at bus 1 even if there are no losses in the connecting branch. It shouldn't matter where the next generation is located.
- f. The locational signal favours bus 1 over bus 2 even when there is up to 0.5 p.u. flow from bus 1 to bus 2, and any increase in generation at bus 1 would increase losses while any increase in generation at bus 2 would reduce losses.
- g. The locational signal shows a dependency on the size of unit at bus 2.

The constant correction factor that was added to the raw loss factors to achieve the load flow loss factors of Figure 2 and Figure 10 is shown in Figure 12. Observations of note are:

- a. The largest correction factor required occurred with 200 MW of load at bus 1 and 200 MW of generation at bus 2.
- b. The magnitude of the largest correction factor is large for the condition with a 200 MW unit at bus 2 and 50 MW flow from bus 2 to bus 1. The largest correction factor is 1.3% or 130% of the system losses of 1% for that condition. The total losses recovered using raw loss factors is actually less than zero (credits are greater in magnitude than charges).
- c. The correction factor at zero power transfer is as high as 1.2%.

Loss factors determined with the incremental method are more sensitive to system changes. Volatility curves based on changes to circuit resistance, load at bus 1 and generation at bus 1 are shown in Figure 13, Figure 14 and Figure 15 respectively. All three volatility factors are larger in magnitude than the corresponding factors determined with the current methodology.

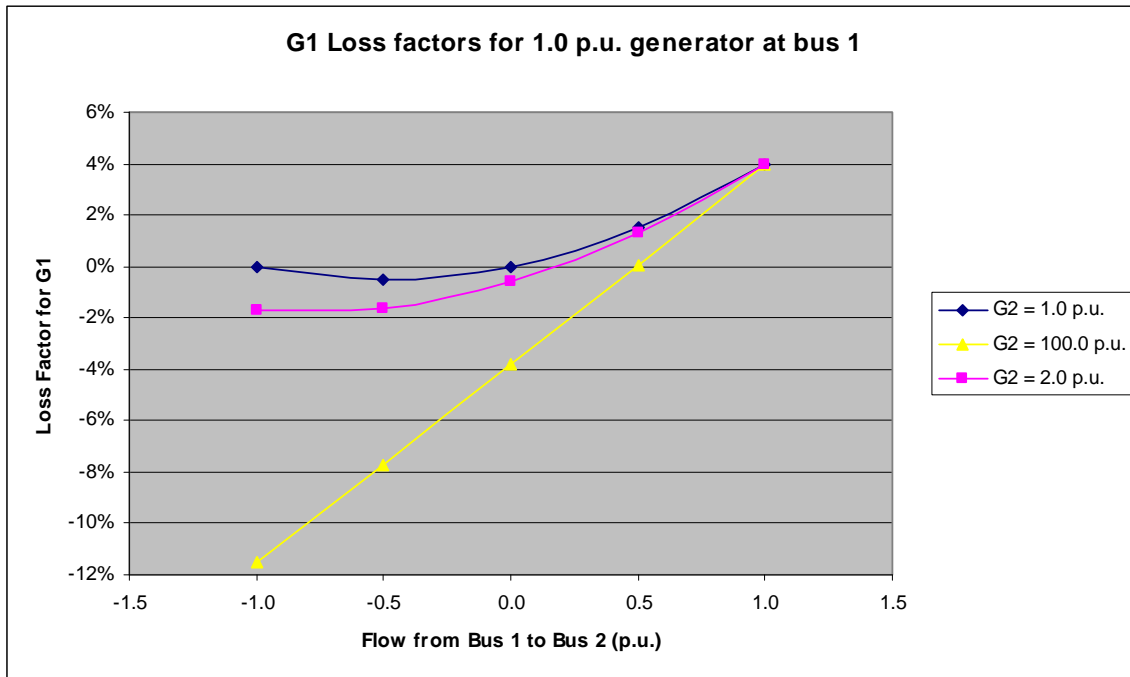


Figure 9: Loss Factors for G1, Incremental Loss Factor Methodology

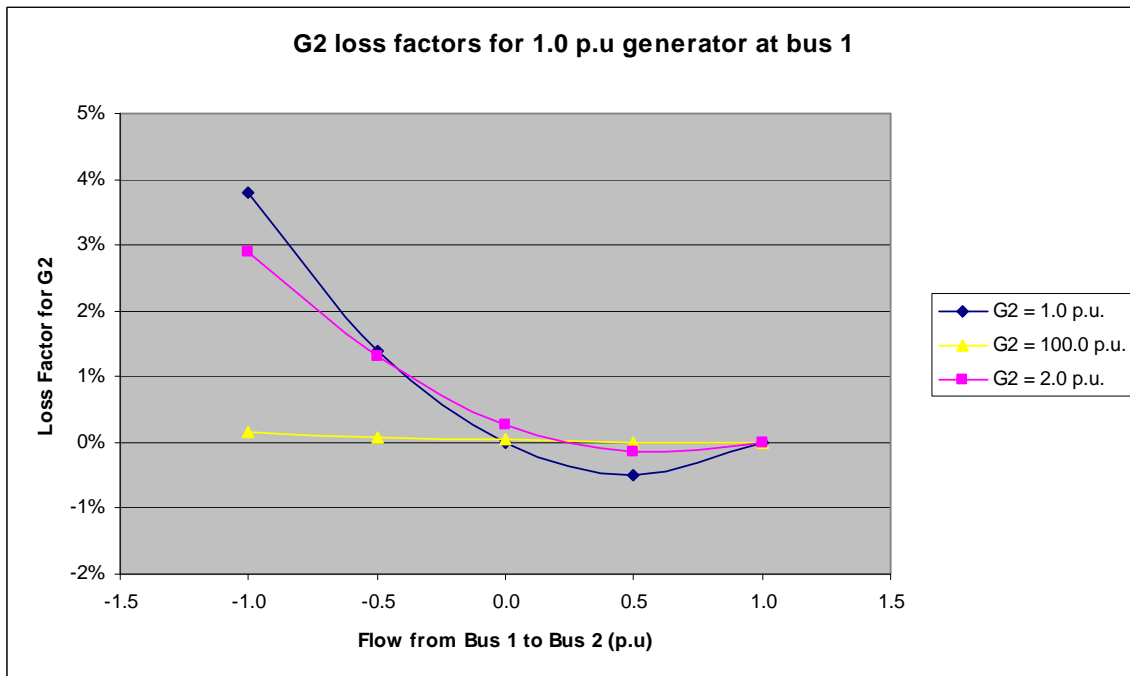


Figure 10: Loss Factors for G2, Incremental Loss Factor Methodology

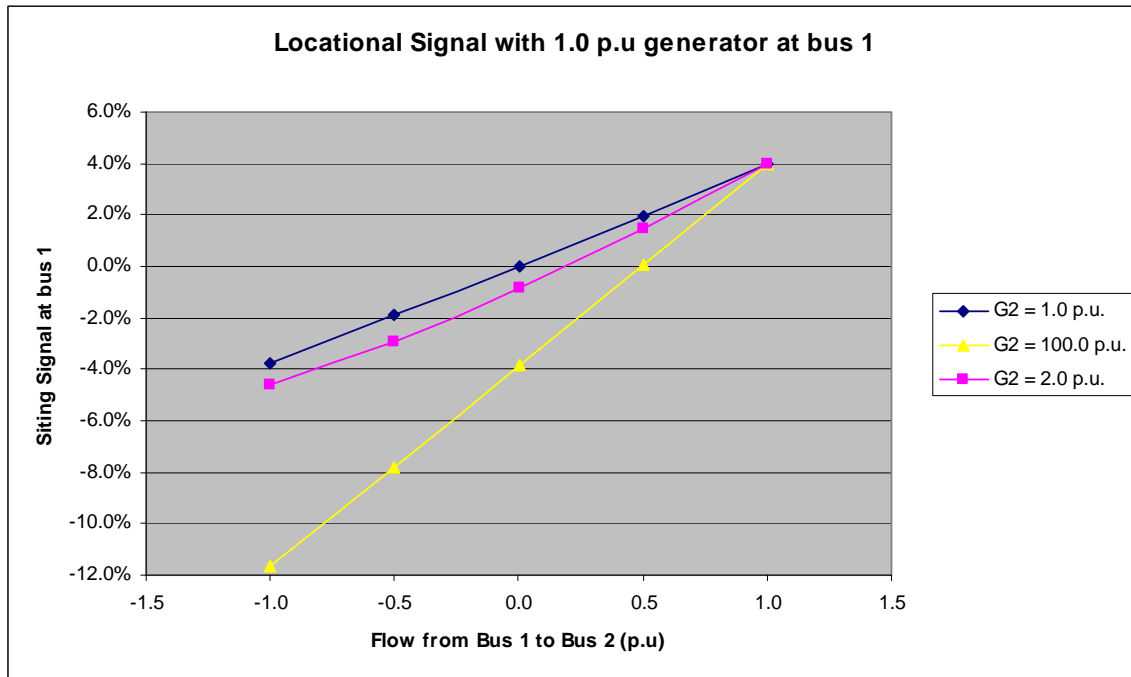


Figure 11: Locational Signal Bus 1 to Bus 2, Incremental Loss Factor Methodology

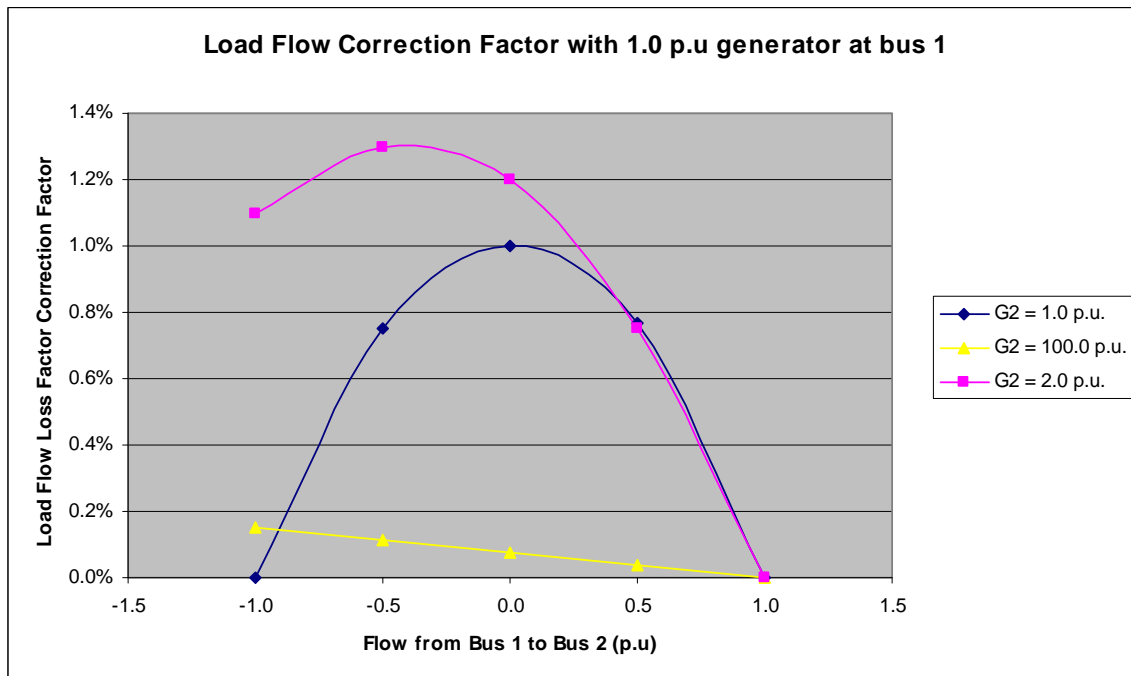


Figure 12: Load Flow Correction Factor, Incremental Loss Factor Methodology

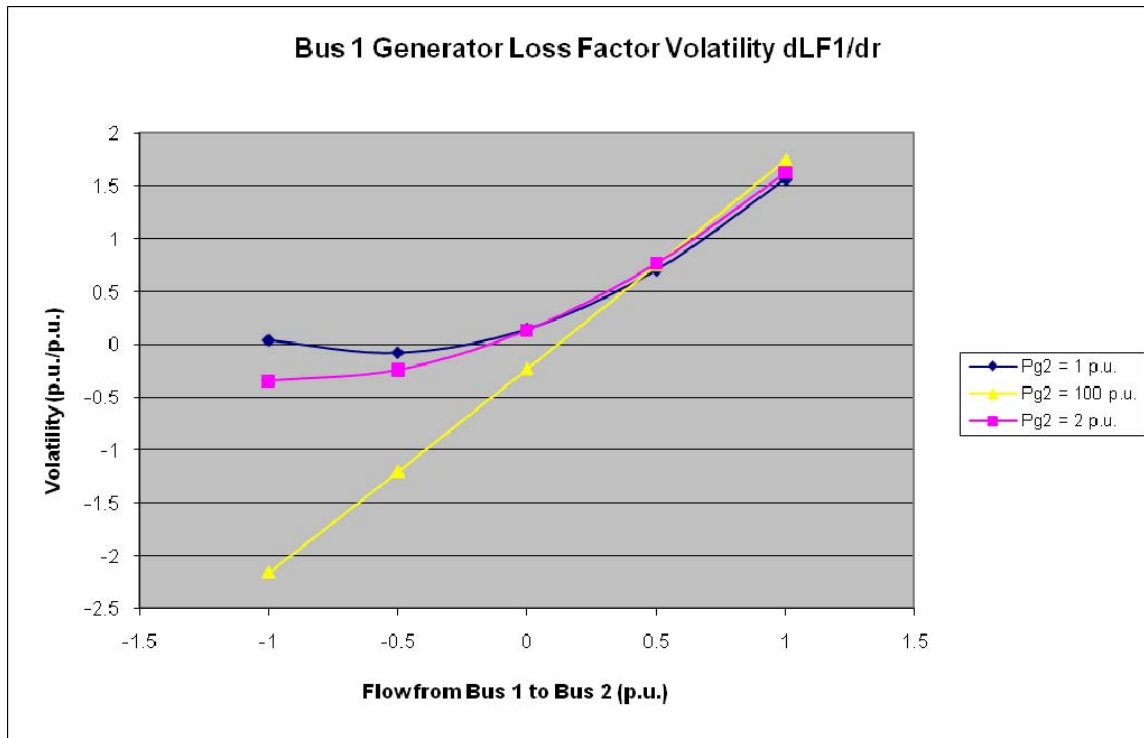


Figure 13: Bus 1 Volatility Based on Resistance Changes, Incremental Loss Factor Methodology

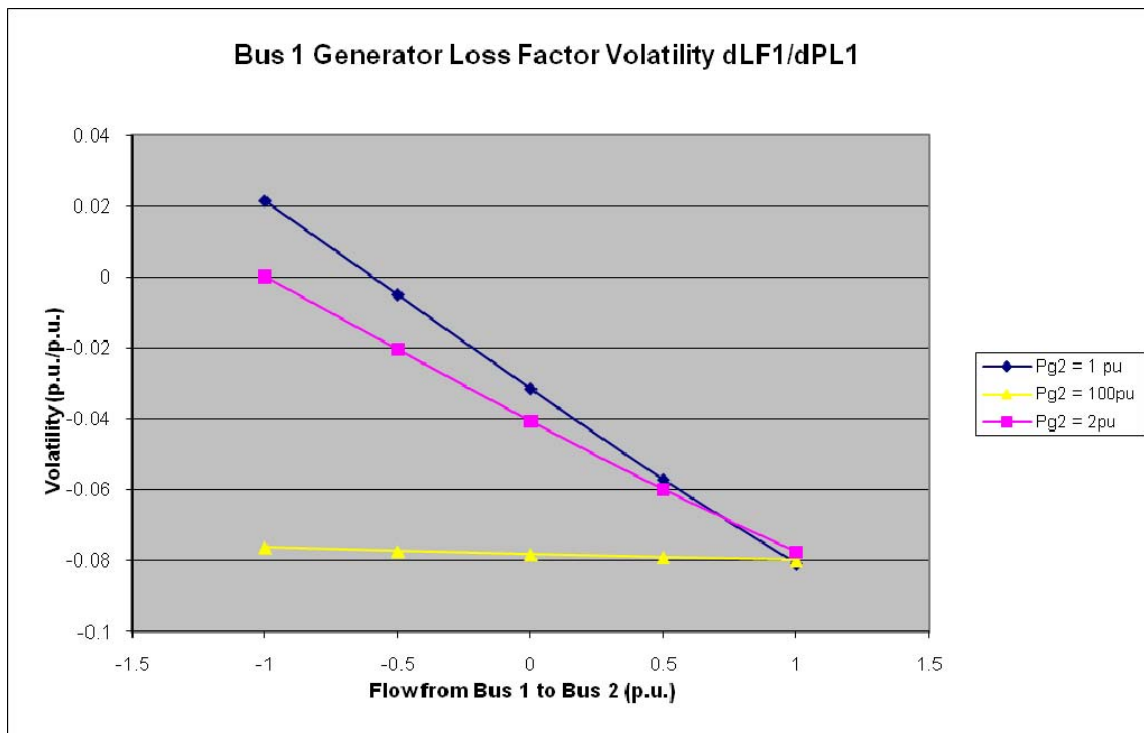


Figure 14: Bus 1 Volatility Based on Bus 1 Load Changes, Incremental Loss Factor Methodology

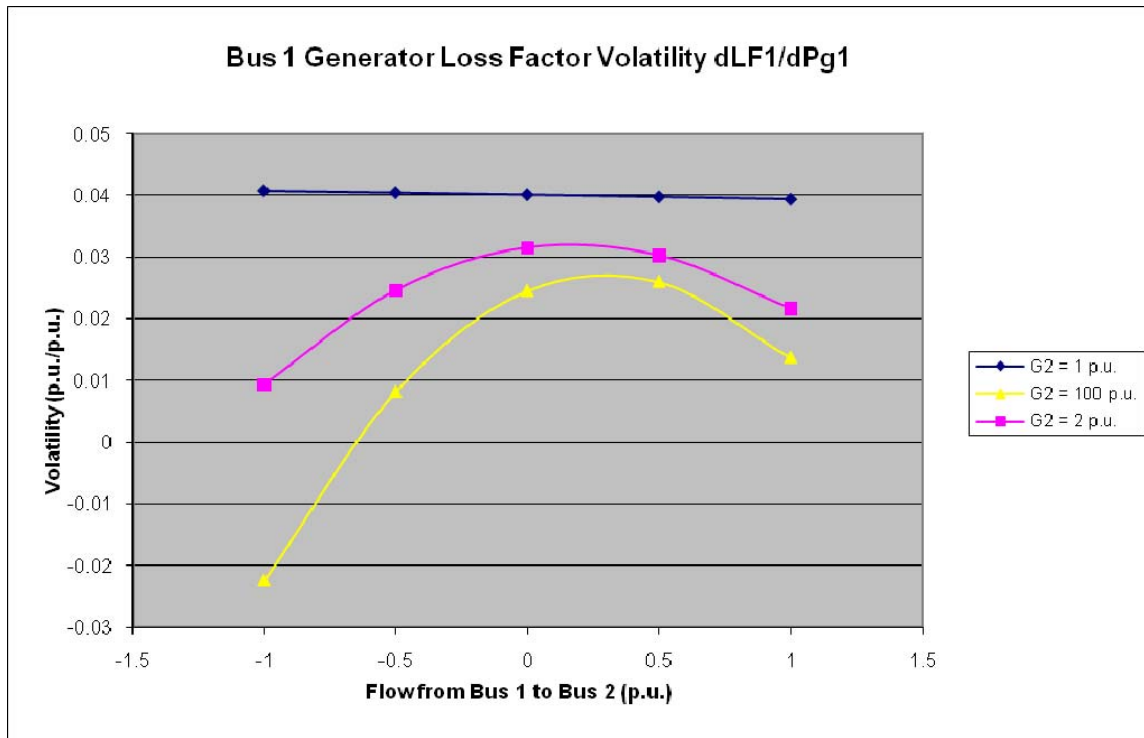


Figure 15: Bus 1 Volatility Based on Bus 1 Generation, Incremental Loss Factor Methodology

5. Other Considerations

If the 100 MW unit at bus 1 is replaced 1 one 25 MW unit and one 75 MW unit, the loss factors at bus1 would remain unchanged for the current methodology, equal to the curves shown in Figure 2 and Figure 3 .

With the incremental loss factor methodology the load flow loss factors would be different for the two units at bus 1 as shown in Figure 16 through Figure 18. The only exception is the one operating condition where the generator output at bus 2 is the same as the total output of the units at bus 1 and the load at bus 2 is zero. This creates a situation where conflicting locational signals can exist. For the condition where the generator at bus 2 has twice the output of the total output at bus 1 (Figure 17) with no flow on the circuit, the 0.25 p.u. unit at bus 1 would receive a credit while the .75 p.u. unit at the same bus would receive a charge.

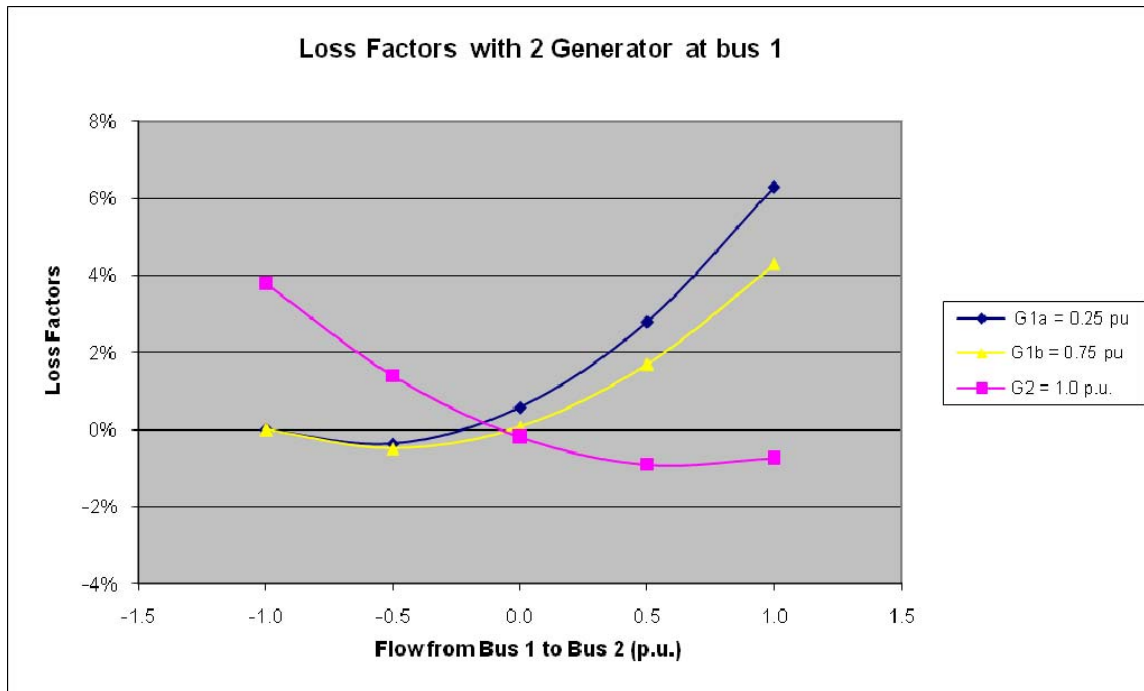


Figure 16 Load Flow Loss Factors for Situation with Different Size Units at Bus 1, Generator at Bus 2 is 1.0 p.u., Incremental Loss Factor Methodology

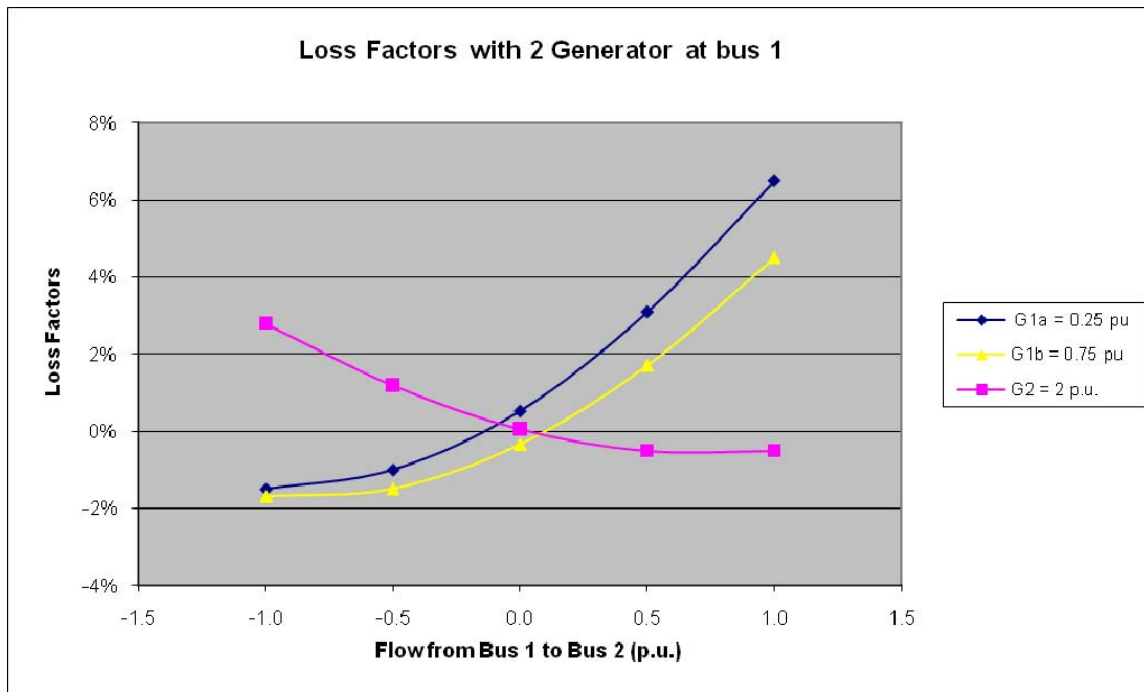


Figure 17 Load Flow Loss Factors for Situation with Different Size Units at Bus 1, Generator at Bus 2 is 2.0 p.u., Incremental Loss Factor Methodology

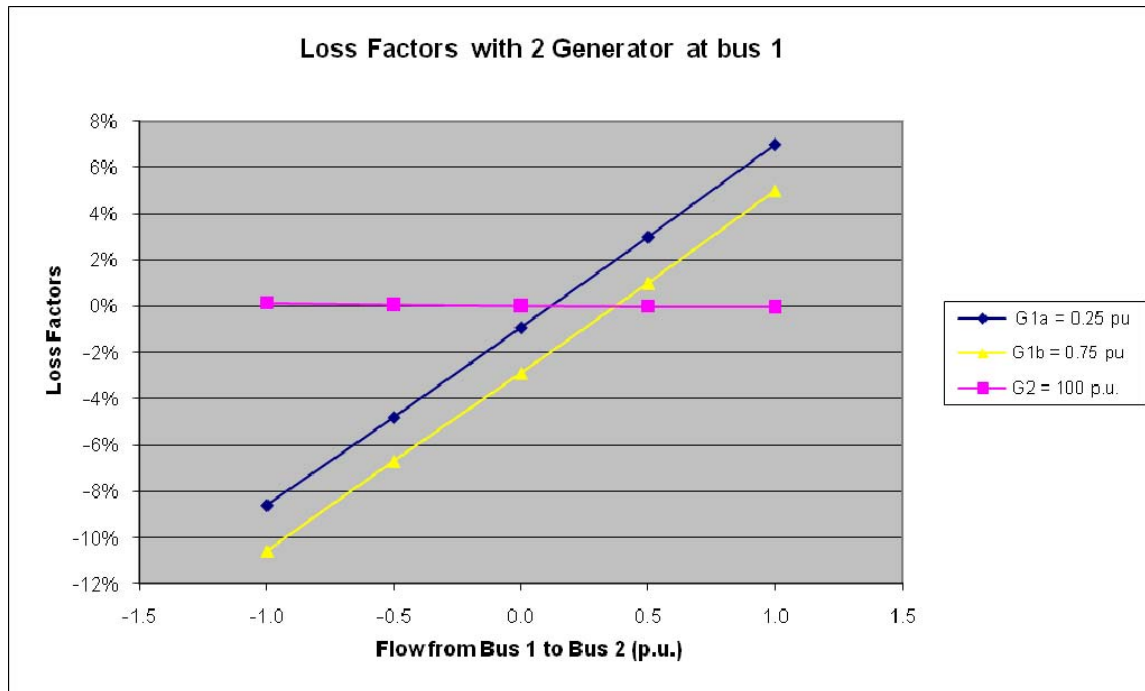


Figure 18 Load Flow Loss Factors for Situation with Different Size Units at Bus 1, Generator at Bus 2 is 100 p.u, Incremental Loss Factor Methodology

6. Comparison of Current Methodology to ILF Methodology

The table below (Table 1) summarizes the differences in loss factor results obtained with the current methodology and the incremental loss factor methodology.

Table 1 Comparison of Methodologies

Comparison	Current Methodology	Incremental Loss Factor
Does the methodology correctly account for losses when there are no losses in the system?	Yes, if there are no losses on the system, neither generator receives a credit or charge. (Figure 2 and Figure 3).	No, the smaller unit could receive a credit and the larger unit a charge. (Figure 9 and Figure 10)
Does the methodology give a logically correct locational signal if there is no flow hence no losses on the circuit?	Yes, the locational signal is zero. (Figure 4)	No the locational signal favours the smaller unit. (Figure 11)

Comparison	Current Methodology	Incremental Loss Factor
Is the locational signal logically correct when system losses are not zero?	Yes, the locational signal favours the bus where there is a shortage of generation (excess of load). (Figure 4)	No, there are some dispatch conditions where the locational signal favours a bus where generation exceeds its local load. (Figure 11)
Does the methodology give the same loss factors if different sized units are connected to the same bus?	Yes, the methodology is based on the total infeed/load balance into the common bus. (Section 5)	No, under some conditions, one generator at the common bus could receive a credit while the other receives a charge. This would create mixed locational signals. Two generators at the same bus would only receive the same loss factor if their output were the same. (Figure 16, Figure 17 & Figure 18)
Are the loss factors sensitive to circuit characteristics and flows on the system ?	Yes system losses are affected by circuit resistance and flows on the circuit so are dependent on net generation and load at each bus. (Figure 6, Figure 7 & Figure 8)	Yes, the effect of system losses and circuit flows is 50% greater with the Incremental Loss Factor method than the current methodology. (Figure 13, Figure 14 & Figure 15)
Is the load flow correction factor small?	Yes. The largest required load flow correction factor for the conditions studied was 0.83% of the system losses. Raw loss factors that are calculated with the 50% area load methodology recover over 99% of the load flow losses. (Figure 5)	No the largest required load flow correction factor is 130% of system losses. For this case, the sum of the products of raw loss factors and generator output is actually negative. (Figure 12)

7. Conclusions

Based on the results presented hereinbefore it is concluded that

The current methodology (50% area) provides logical results in the two bus system for the range of generation/load scenarios suggested by the Milner representatives as well as the situation where generation at both ends of the circuit has the same output.

The incremental loss factor methodology provides illogical results for the range of generation/load scenarios suggested by the Milner representatives. It only meets criteria for the very limited situation where generation at both ends of the circuit has the same output.

8. References

[1] Report "Alberta Electric System Operator, Loss Factor Methodologies Evaluation, Part 1 - Determination of 'Raw' Loss Factors and Load Flow Shift Factors" Prepared by Teshmont Consultants LP, Revised 2007, May 22.

Appendix A

MathCAD worksheet equations for Calculating Loss Factors for 50% Area Load Methodology

Calculate Circuit Flow (power and current) at Bus 1 From pg_1 , pl_1 , and v_1

$$p_1 := pg_1 - pl_1$$

$$i_1 := \frac{p_1}{v_1}$$

Calculate Circuit Flow at Bus 2

$$i_2 := -i_1$$

$$p_2 := -p_1 + (i_1)^2 \cdot r$$

Calculate bus 2 voltage

$$v_2 := \begin{cases} v_1 & \text{if } i_2 = 0 \\ \frac{p_2}{i_2} & \text{otherwise} \end{cases}$$

Calculate load at Bus 2

$$pl_2 := pg_2 - p_2$$

Circuit Losses

$$j_w := \left(\frac{p_1}{v_1} \right)^2 \cdot r$$

Total Load

$$p_{tot} := pl_1 + pl_2$$

Fraction of total load at bus 1 and Bus 2

$$k_1 := \frac{pl_1}{p_{tot}}$$

$$k_2 := \frac{pl_2}{p_{tot}}$$

Raw Loss Factor at Bus 1

$$L_{fraw_1} := \left(\frac{p_1}{v_1^2} \right) \cdot r \cdot \frac{-k_2}{\left[k_1 \cdot \left(2 \cdot \frac{p_1}{v_1^2} \right) \cdot r - k_1 - k_2 \right]}$$

Raw Loss Factor at Bus 2

$$L_{fraw_2} := \left(\frac{p_2}{v_2^2} \right) \cdot r \cdot \frac{-k_1}{\left[k_2 \cdot \left(2 \cdot \frac{p_2}{v_2^2} \right) \cdot r - k_2 - k_1 \right]}$$

Load Flow Correction Factor

$$sf := \frac{1 - L_{fraw_1} \cdot pg_1 - L_{fraw_2} \cdot pg_2}{pg_1 + pg_2}$$

final Loss Factors

$$lf_1 := L_{fraw_1} + sf$$

$$lf_2 := L_{fraw_2} + sf$$

Appendix B

MathCAD worksheet equations for Calculating Incremental Loss Factor Methodology

(Two generators at Bus 1)

Total Generation at Bus 1

$$pg_1 := pga_1 + pgb_1$$

Losses with pga_1 set to zero

$$\alpha_{a1} := \left[\begin{array}{l} \frac{pg_2 + pgb_1 - \left(\frac{pgb_1}{v_1}\right)^2 \cdot r}{pl_2} \text{ if } pl_1 = 0 \\ \frac{\left[-\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pgb_1 \cdot \frac{r}{v_1} \right) + \sqrt{\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pgb_1 \cdot \frac{r}{v_1} \right)^2 - 4 \cdot \left(\frac{pl_1}{v_1} \right)^2 \cdot \left[-pg_2 - pgb_1 + \left(\frac{pgb_1}{v_1} \right)^2 \cdot r \right]}{2 \cdot \left(\frac{pl_1}{v_1} \right)^2 \cdot r} \right]}{\text{otherwise}} \end{array} \right]$$

$$lla1_0 := \left(\frac{pgb_1 - pl_1 \cdot \alpha_{a1}}{v_1} \right)^2 \cdot r$$

$$lfaraw_1 := \frac{1 - lla1_0}{pga_1}$$

Losses with pgb_1 set to zero

$$\alpha_{b1} := \left[\begin{array}{l} \frac{pg_2 + pga_1 - \left(\frac{pga_1}{v_1}\right)^2 \cdot r}{pl_2} \text{ if } pl_1 = 0 \\ \frac{\left[-\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pga_1 \cdot \frac{r}{v_1} \right) + \sqrt{\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pga_1 \cdot \frac{r}{v_1} \right)^2 - 4 \cdot \left(\frac{pl_1}{v_1} \right)^2 \cdot \left[-pg_2 - pga_1 + \left(\frac{pga_1}{v_1} \right)^2 \cdot r \right]}{2 \cdot \left(\frac{pl_1}{v_1} \right)^2 \cdot r} \right]}{\text{otherwise}} \end{array} \right]$$

Losses with pgb_1 set to zero

$$\alpha_{b1} := \left[\begin{array}{l} \frac{pg_2 + pga_1 - \left(\frac{pga_1}{v_1}\right)^2 \cdot r}{pl_2} \text{ if } pl_1 = 0 \\ \frac{\left[-\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pga_1 \cdot \frac{r}{v_1} \right) + \sqrt{\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pga_1 \cdot \frac{r}{v_1} \right)^2 - 4 \cdot \left(\frac{pl_1}{v_1}\right)^2 \cdot r \cdot \left[-pg_2 - pga_1 + \left(\frac{pga_1}{v_1}\right)^2 \cdot r \right]} \right]}{2 \cdot \left(\frac{pl_1}{v_1}\right)^2 \cdot r} \text{ otherwise} \end{array} \right]$$

$$11b1_0 := \left(\frac{pga_1 - pl_1 \cdot \alpha_{b1}}{v_1} \right)^2 \cdot r$$

$$1fbraw_1 := \begin{cases} 0 & \text{if } pgb_1 = 0 \\ \frac{1 - 11b1_0}{pgb_1} & \text{otherwise} \end{cases}$$

Losses with pg_2 set to zero

$$\alpha_2 := \left[\begin{array}{l} \frac{pg_1 - \left(\frac{pg_1}{v_1}\right)^2 \cdot r}{pl_2} \text{ if } pl_1 = 0 \\ \frac{\left[-\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pg_1 \cdot \frac{r}{v_1} \right) + \sqrt{\left(pl_1 + pl_2 - 2 \cdot pl_1 \cdot pg_1 \cdot \frac{r}{v_1} \right)^2 - 4 \cdot \left(\frac{pl_1}{v_1}\right)^2 \cdot r \cdot \left[-0 - pg_1 + \left(\frac{pg_1}{v_1}\right)^2 \cdot r \right]} \right]}{2 \cdot \left(\frac{pl_1}{v_1}\right)^2 \cdot r} \text{ otherwise} \end{array} \right]$$

$$112_0 := \left(\frac{pg_1 - pl_1 \cdot \alpha_2}{v_1} \right)^2 \cdot r$$

$$1fraw_2 := \begin{cases} 0 & \text{if } pg_2 = 0 \\ \frac{1 - 112_0}{pg_2} & \text{otherwise} \end{cases}$$

$$sf_{1f} := \frac{1 - (lfaraw_1 \cdot pg_{a1} + lfbraw_1 \cdot pg_{b1} + lfraw_2 \cdot pg_2)}{pg_1 + pg_2}$$

$$lfilfa_1 := lfaraw_1 + sf_{1f}$$

$$lfilfb_1 := lfbraw_1 + sf_{1f}$$

$$lfilf_2 := lfraw_2 + sf_{1f}$$